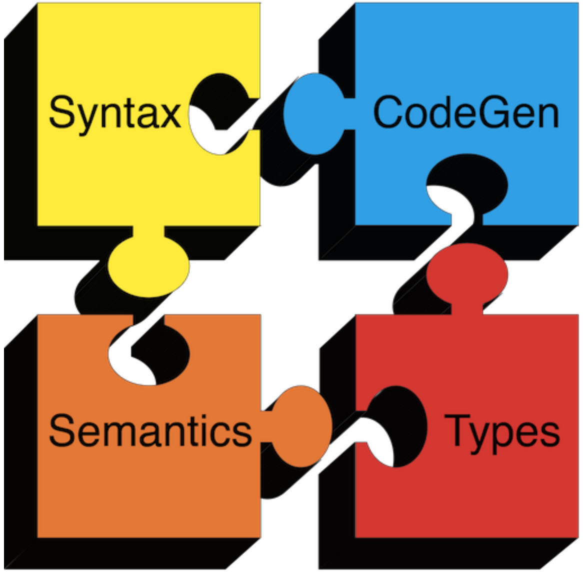


Compilers

Graph Coloring

Graph Coloring

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors

- A graph is k-colorable if it has a coloring with k colors

Graph Coloring

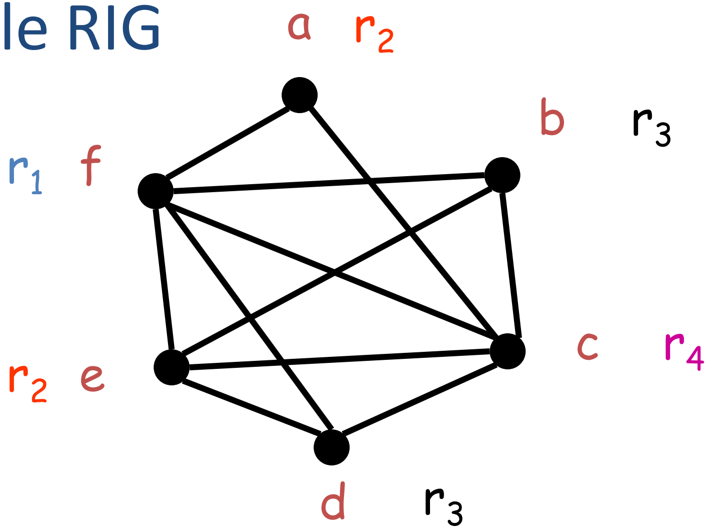
- In our problem, colors = registers
  - We need to assign colors (registers) to graph nodes (temporaries)

- Let k = number of machine registers

- If the RIG is k-colorable then there is a register assignment that uses no more than k registers

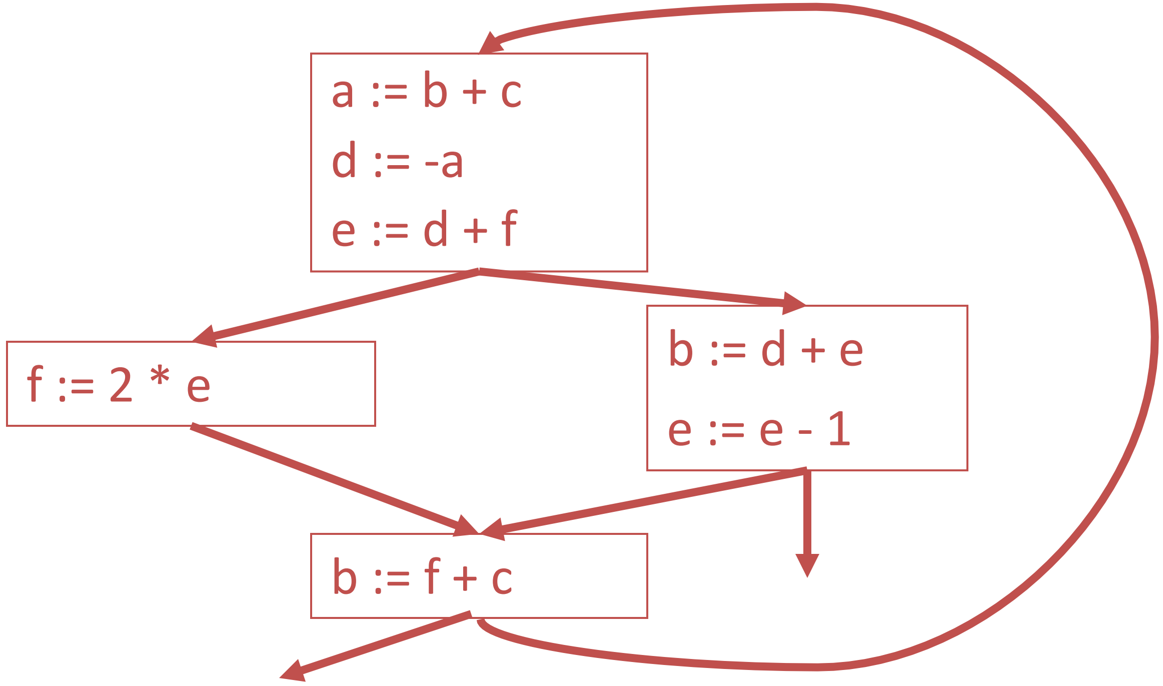
Graph Coloring

- Consider the example RIG



- There is no coloring with less than 4 colors
- There are 4-colorings of this graph

Graph Coloring



```
a := b + c
d := -a
e := d + f
```

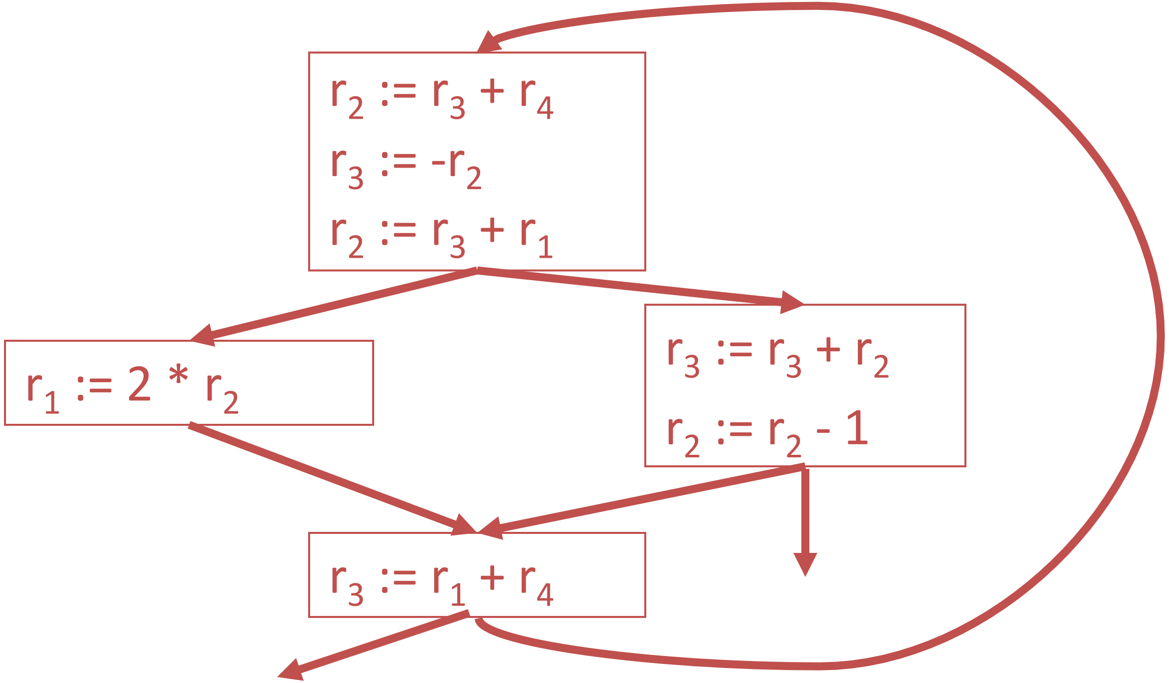
```
f := 2 * e
```

```
b := d + e
e := e - 1
```

```
b := f + c
```

Alex Aiken

Graph Coloring



$r_2 := r_3 + r_4$

$r_3 := -r_2$

$r_2 := r_3 + r_1$

$r_1 := 2 * r_2$

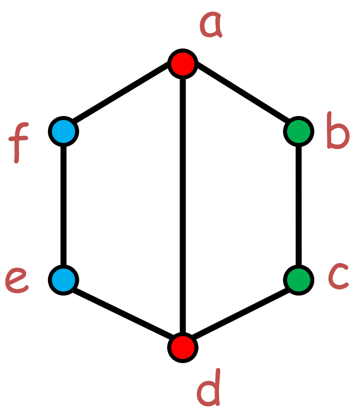
$r_3 := r_3 + r_2$

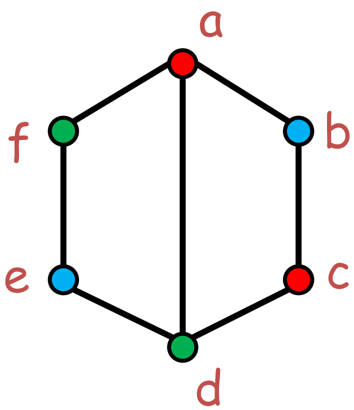
$r_2 := r_2 - 1$

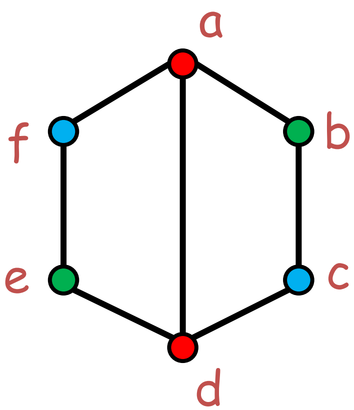
$r_3 := r_1 + r_4$

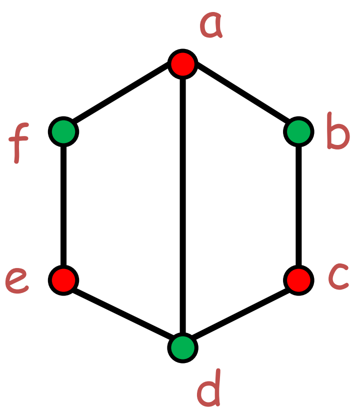
Graph Coloring

Which of the following colorings is a valid minimal coloring of the given RIG?

○


○


○


○


Graph Coloring

- How do we compute graph colorings?

- It isn’t easy:

1. This problem is very hard (NP-hard). No efficient algorithms are known.
   - *Solution: use heuristics*
2. A coloring might not exist for a given number of registers
   - *Solution: later*

Graph Coloring

- Observation:
  - Pick a node t with fewer than k neighbors in RIG
  - Eliminate t and its edges from RIG
  - If resulting graph is k-colorable, then so is the original graph

- Why?
  - Let $c_1,\ldots,c_n$ be the colors assigned to the neighbors of t in the reduced graph
  - Since $n < k$ we can pick some color for t that is different from those of its neighbors

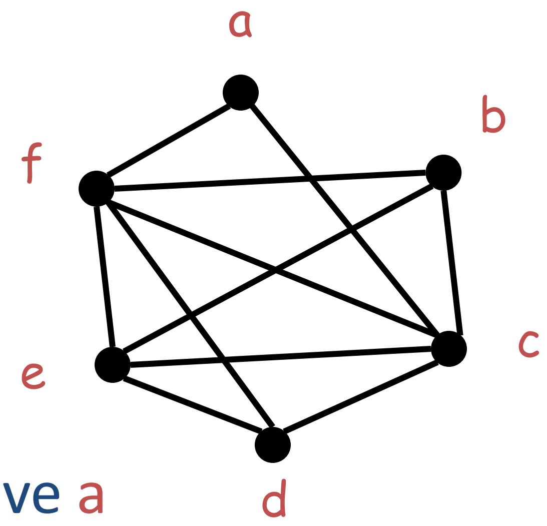
Graph Coloring

- The following works well in practice:
  - Pick a node t with fewer than k neighbors
  - Put t on a stack and remove it from the RIG
  - Repeat until the graph is empty

- Assign colors to nodes on the stack
  - Start with the last node added
  - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring

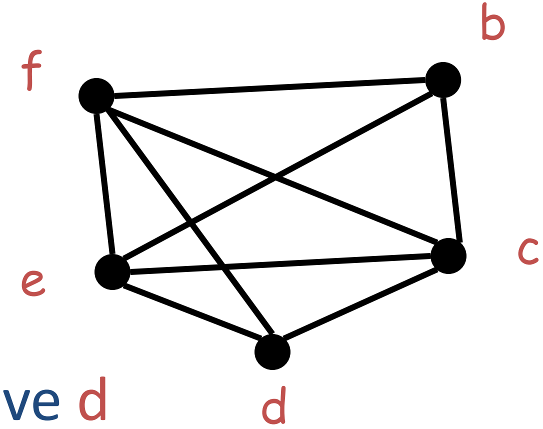
- Start with the RIG and with k = 4:



Stack: {}

- Remove a

Graph Coloring

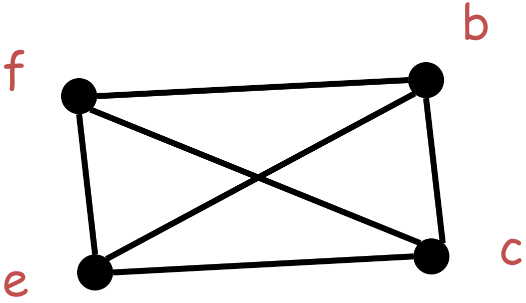


Stack: {a}

- Remove d

Graph Coloring

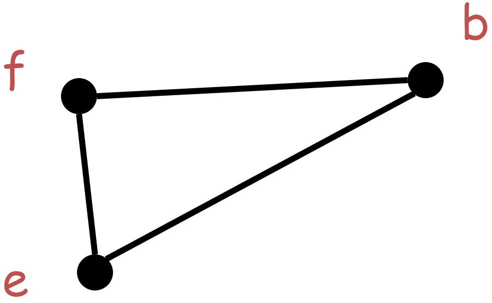
- Note: all nodes now have fewer than 4 neighbors



Stack: {d, a}

- Remove c

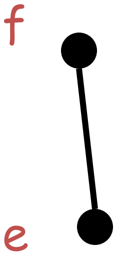
Graph Coloring



Stack: {c, d, a}

- Remove b

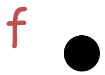
Graph Coloring



Stack: {b, c, d, a}

- Remove e

Graph Coloring



Stack: {e, b, c, d, a}

- Remove f

Graph Coloring

Stack: {f, e, b, c, d, a}

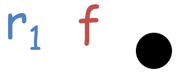
- Empty graph – done with the first part!

Graph Coloring

- Now start assigning colors to nodes, starting with the top of the stack

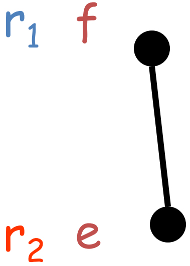
Stack: {f, e, b, c, d, a}

Graph Coloring



Stack: {e, b, c, d, a}

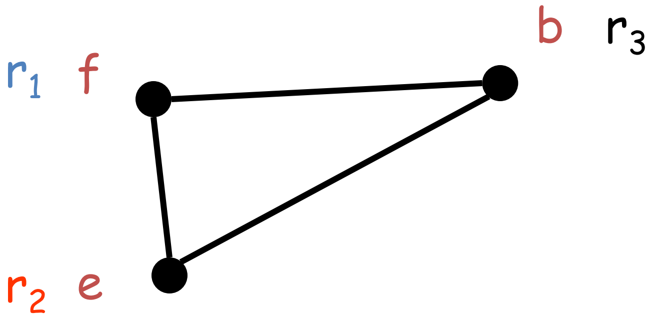
Graph Coloring



Stack: {b, c, d, a}

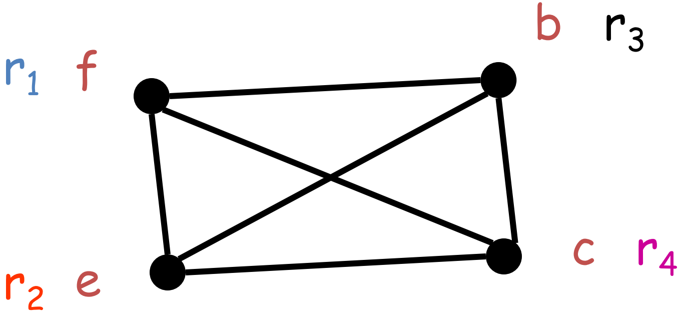
- e must be in a different register from f

Graph Coloring



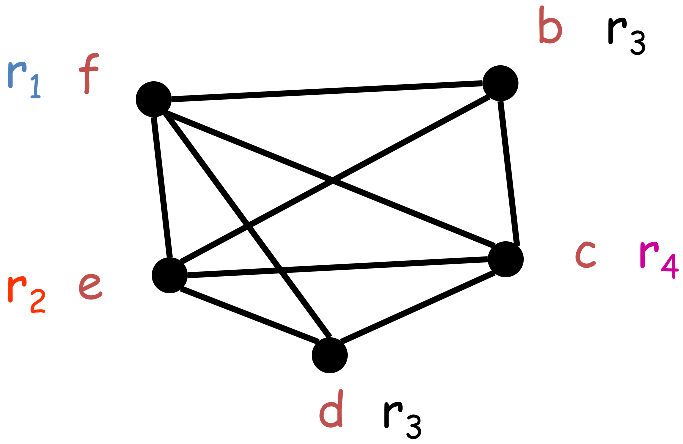
Stack: {c, d, a}

Graph Coloring



Stack: {d, a}

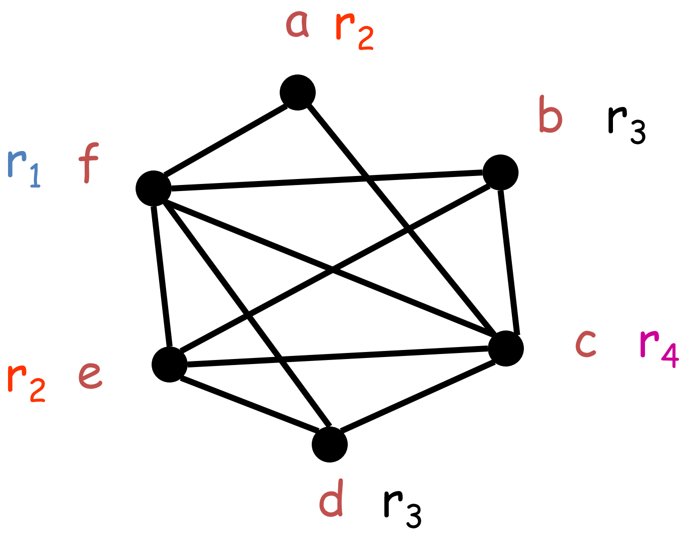
Graph Coloring



Stack: {a}

- d can be in the same register as b

Graph Coloring



Stack: {}

Graph Coloring

For the given RIG and k = 3, which of the following are valid deletion orders for the nodes of the RIG?

○ {d, e, c, b, a, f}

○ {e, f, a, b, c, d}

○ {d, c, b, a, f , e}

○ {d, e, b, c, a, f}