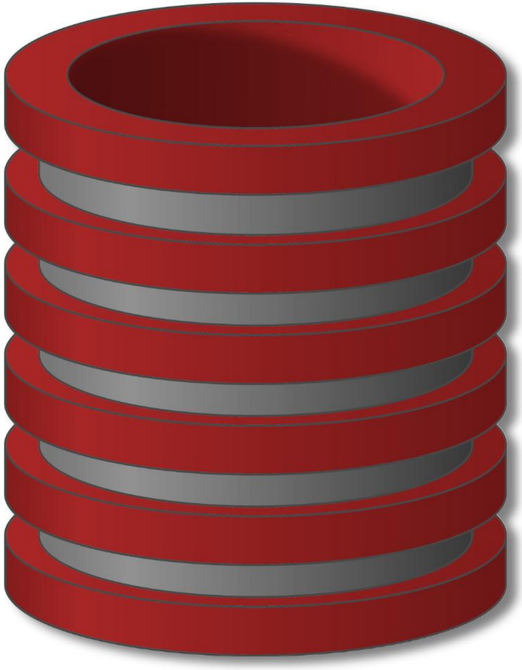


Relational Design Theory

Functional Dependencies

Jennifer Widom

Relational design by decomposition

- "Mega" relations + properties of the data
- System decomposes based on properties
- Final set of relations satisfies normal form
  - No anomalies, no lost information
- Functional dependencies ⇒ Boyce-Codd Normal Form
- Multivalued dependences ⇒ Fourth Normal Form

Functional dependencies are generally useful concept

- Data storage – compression
- Reasoning about queries – optimization

Keys

Example: College application info.

```
Student(SSN, sName, address,
        HScode, HSname, HScity, GPA, priority)

Apply(SSN, cName, state, date, major)
```

`Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)`

Suppose `priority` is determined by `GPA`

$GPA > 3.8$ priority $= 1$

$3.3 < GPA \leq 3.8$ " $= 2$

$GPA \leq 3.3$ " $= 3$

Two tuples with same `priority` have same `GPA`

Functional Dependencies

**Student(SSN, sName, address,
HScode, HSname, HScity, GPA, priority)**

Two tuples with same **priority** have same **GPA**

$\forall\, t, u \in R$:

$t[A_1, .. A_n] = u[A_1, ... A_n] \Rightarrow t[B_1, .. B_m] = u[B_1, ... B_n]$

R

$$\underbrace{A_1, A_2, ..., A_n}_{\bar{A}} \rightarrow \underbrace{B_1, B_2, ..., B_m}_{\bar{B}}$$

Functional Dependency

- Based on knowledge of real world
- All instances of relation must adhere

$\boxed{\bar{A} \rightarrow \bar{B}}$

$R(\bar{A}, \bar{B}, \bar{C})$

$\bar{A}$	$\bar{B}$	$\bar{C}$
$\bar{a}$	$\bar{b}$	$\bar{c}_1$
$\bar{a}$	$\bar{b}$	$\bar{c}_2$

Functional Dependencies

Student(SSN, sName, address, HScode, HSname, HScity, GPA, priority)

SSN → sName

SSN → address

HScode → HSname, HScity

HSname, HScity → HScode

SSN → GPA

GPA → priority

SSN → priority

more

Apply(SSN, cName, state, date, major)

cName → date

SSN, cName → major

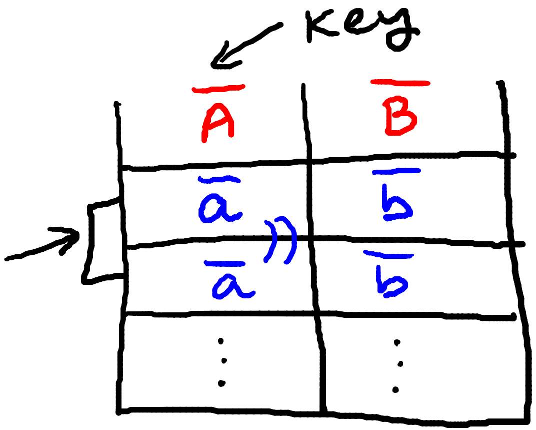
SSN → state

Functional Dependencies and Keys

- Relation with no duplicates
- Suppose $\bar{A} \rightarrow$ all attributes

$R(\bar{A}, \bar{B})$

Key



Trivial Functional Dependency

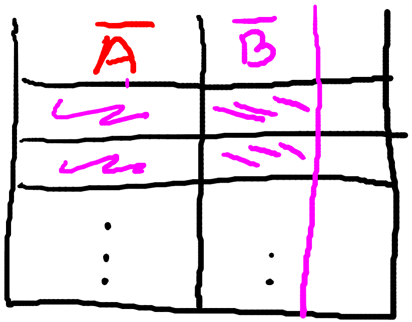
$\bar{A} \rightarrow \bar{B}$ $\quad \bar{B} \subseteq \bar{A}$

Nontrivial FD

$\bar{A} \rightarrow \bar{B}$ $\quad \bar{B} \nsubseteq \bar{A}$

Completely nontrivial FD

$\bar{A} \rightarrow \bar{B}$ $\quad \bar{A} \cap \bar{B} = \emptyset$



Rules for Functional Dependencies

Splitting rule

$\bar{A} \rightarrow B_1, B_2, \ldots, B_m$

$\Rightarrow \bar{A} \rightarrow B_1 \quad \bar{A} \rightarrow B_2 \quad \ldots$

Can we also split left-hand-side?

$A_1, A_2, \ldots, A_n \rightarrow \bar{B}$

? $A_1 \rightarrow \bar{B} \quad A_2 \rightarrow \bar{B}$

No

HSname, HScity → HScode

HSname → HScode

Rules for Functional Dependencies

Combining rule

$$\bar{A} \rightarrow B_1$$

$$\bar{A} \rightarrow B_2$$

$$\vdots$$

$$\bar{A} \rightarrow B_n$$

$$\Rightarrow \quad \bar{A} \rightarrow B_1, \ldots, B_n$$

Rules for Functional Dependencies

Trivial-dependency rules

$\bar{A} \rightarrow \bar{B}$ $\quad \bar{B} \subseteq \bar{A}$

$\bar{A} \rightarrow \bar{B}$ then $\bar{A} \rightarrow \bar{A} \cup \bar{B}$

$\bar{A} \rightarrow \bar{B}$ then $\bar{A} \rightarrow \bar{A} \cap \bar{B}$

splitting

Rules for Functional Dependencies

Transitive rule

$\bar{A} \to \bar{B}$ $\quad \bar{B} \to \bar{C}$

then $\bar{A} \to \bar{C}$

$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{D}$
$\bar{a}$	$\bar{b}$	$\bar{c}$	
$\bar{a}$	$\bar{b}$	$\bar{c}$	
⋮	⋮	⋮	⋮

Closure of Attributes

- Given relation, FDs, set of attributes $\bar{A}$
- Find all B such that $\bar{A} \rightarrow B$

$\bar{A}^+$ $\{A_1, \ldots, A_n\}^+$

$A_4 \rightarrow CD$
$C \rightarrow E$

start with $\{A_1, \ldots, A_n, C, D, E\}$

repeat until no change:

if $\bar{A} \rightarrow \bar{B}$ and $\bar{A}$ in set

add $\bar{B}$ to set

Closure Example

```
Student(SSN, sName, address,
            HScode, HSname, HScity, GPA, priority)
```

```
SSN → sName, address, GPA
GPA → priority
HScode → HSname, HScity
```

$\{SSN, HScode\}^+$ → all attrs. Key

{SSN, HScode, sName, address,
GPA, priority, HSname, HScity}

Closure and Keys

Is $\bar{A}$ a key for R ? → FDs

Compute $\bar{A}^+$ If = all attrs
then $\bar{A}$ is a key.

How can we find all keys given a set of FDs?

C Consider every subset of attrs
↳ $\bar{A}$

$\bar{A}^+$ → all attrs
key

↑
increasing size
AB → all attrs

Specifying FDs for a relation

- S_1 and S_2 sets of FDs
- S_2 "follows from" S_1 if every relation instance satisfying S_1 also satisfies S_2

S_2: { SSN → priority }

S_1: { SSN → GPA, GPA → priority }

How to test?

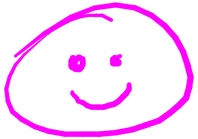
Does $A \rightarrow B$ follow from S ? S_1 S_2

(1) $\bar{A}^+$ based on S check if $\bar{B}$ in set.

(2) Armstrong's Axioms

Specifying FDs for a relation

Want: *Minimal set of completely nontrivial FDs such that all FDs that hold on the relation follow from the dependencies in this set*



Functional dependencies are generally useful concept

- Relational design by decomposition

  Functional dependencies $\Rightarrow$ Boyce-Codd Normal Form

- Data storage – compression
- Reasoning about queries – optimization